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Abstract

OECD countries at large are concerned with strong male-female disparities in the labour market, in particular: with a wage gap in favour of men; with a strong gender occupational segmentation. In this paper, we propose an employment relationship model within which agents exhibit self-esteem motives. These motives can impact their motivation for work and effort. We rely on this model to develop an explanation of observed disparities. Consistent with the empirical evidence, the model gives an account of a vertically segmented labour market, that is to say of an overrepresentation of men in better-paid jobs; an overrepresentation of women in poorly-paid jobs.

Key words: *Employment relationship, work motivation, self-esteem, vertical segregation of the labour market.*

ÉCART HOMME-FEMME DE RÉMUNERATION MOYENNE ET SEGMENTATION PROFESSIONNELLE VERTICALE : LE RÔLE DE LA MOTIVATION AU TRAVAIL

RESUME

L'ensemble des pays de l'OCDE est marqué par de fortes disparités homme-femme sur le marché du travail, notamment : par un écart de rémunération moyenne favorable aux hommes ; par une forte segmentation professionnelle par genre. Dans cet article, nous proposons un modèle de relation d'emploi au sein duquel les agents manifestent des motifs d'estime de soi. Ces motifs se répercutent sur leur motivation au travail. Nous nous appuyons sur ce modèle pour développer une explication des disparités observées. Conformément aux faits, le modèle rend compte d'une segmentation verticale du marché du travail, c'est-à-dire d'une surreprésentation des hommes parmi les emplois les mieux rémunérés ; une surreprésentation des femmes parmi les emplois les moins rémunérés.

Mots-clefs : Relation d’emploi, motivation au travail, estime de soi, segmentation verticale du marché du travail.

According to a report from the OECD,[1] for thirteen European countries,[2] on average, once the effects of education, tenure, potential experience and other observable characteristics are controlled for, gross hourly wages are still 15% higher for men than for women. This gap can be imputed to two sources: unobserved differences in gender characteristics and/or wage discrimination against women. Male-female segmentation of the labour market being widely documented,[3] notably in its vertical dimension,[4] the authors of the report suspect differences between the jobs mostly held by men or by women to represent a significant source of gender differences in unobserved differences. This suspicion is strengthened by the observation of a strong and significant correlation between the unexplained part of the gender wage gap and a dissimilarity index for each country: occupational segmentation seems to play a large role in the average wage gap between men and women.[5] In contrast, the assumption of a pure pay discrimination against women, *i.e.* a systematically lower female wage for a given job within a given company, does not look so convincing empirically.[6]

If the assumption of a pure pay discrimination is put aside, it appears that an explanation of a male-female wage gap consistent with empirical evidence should require: 1) an account of the gender segmentation of the labour market; 2) (desirably) on explicit presentation of the unobserved individual characteristics involved in the determining of a wage gap. Macpherson and Hirsch (1995, p. 463) state the major question indicating that a crucial point as regards this issue is to understand how and why the labour market sorts women and men into jobs with different characteristics and productivities. The present work aims to provide an explanation of the male-female wage gap which meets previous requisites and answering the question raised by Macpherson et Hirsch.

Among the unobserved individual characteristics that could make a difference between male and female working persons in the labour market, the OECD report mentioned above, points

[1] See *The OECD Employment Outlook* (2002, p.65).

[2] Austria, Belgium, Denmark, Finland, France, Germany, Greece, Irland, Italy, Netherlands, Portugal, Spain, the United-Kingdom.

[3] See *The OECD Employment Outlook* (2002, pp. 85-95).

[4] In the literature, this notion of verticality sometimes conveys hierarchical aspects. In this article, we simply use the expression to describe: an overrepresentation of men in the well-paid jobs; an underrepresentation of women in poorly-paid jobs.

[5] See *The OECD Employment Outlook* (2002, pp. 85-95). This conclusion meets that of Johnson and Solon (1986) who show, for the United-States, that gender wage differences result more from the fact that men are hired in firms which pay well compared to those where women predominate than from pure pay discrimination within given firms. See Blau and Kahn (2002) for more references.

[6] See, for the United-States, Johnson and Solon (1986) or Macpherson and Hirsch (1995).

out *motivation*: men and women possibly develop a differentiated motivation to work and effort. This assumption is rather natural; in particular it is in line with the evidence provided by a variety of empirical studies - see, for instance, Lambert (1991).[7] Yet, it has had little impact on conventional economic analyses according to which motivation and incentives are inseparable. Indeed, within this perspective, a male-female difference in motivation can only result from a difference in wage incentives: a circular argument. The authors of the OECD report obviously rather refer to non wage forms of work motivation. Many trails have started to be explored on this issue, notably among the proponents of the behavioural approach in economics. Fehr and Falk (2002) provide a variety of experimental data putting forward the relevance, within the framework of an employment relationship of: such motives as the reciprocity, the need for social approval, the observing of social norms; phenomena such as intrinsic motivation. Frey (1997) studies analytically the economic stakes of intrinsic motivation and its sensitivity to incentives. Within a cognitive perspective, Bénabou and Tirole (2006) study the link between self-esteem and motivation through a signalling and self-signalling argument: some actions of agents are aimed at signalling and/or self-signalling an ideal self-image, valued in itself in terms of well-being. This latter work follows the inspiration of the more applied analyses of Akerlof and Kranton (2000, 2005) who directly rely on the conclusions of social psychology using the notion of *identity* as a tool for economics.

In the present paper, we provide an explanation of a socio-demographic segmentation of the labour market (above all, the gender segmentation of the labour market) and of a socio-demographic gap in average earnings (above all, the gender wage gap) originated in employment relationship model which mobilizes self-esteem as a source of non wage motivation at work. To do this, within a perspective close to that of Akerlof and Kranton (2000, 2005), we rely on a standard principal-agent model to which is introduced a trade-off binding the agent's self-esteem motives. This trade-off corresponds for the latter (from now on, *he*) to the choice of a context of self-evaluation, that is to say, of a social context relative to which the agent builds his conception of his self-esteem. We reduce this choice to that of an identity more or less polarized towards working life. When this polarity is strong, a self-esteem motivation adds to the usual pecuniary motivation to determine the agent's sensitivity to incentives. Although the principal (from now on, *she*) may take advantage of this added work motivation, she is constrained by the characteristics of the job she wants to fill; characteristics of which are summarized by: the

[7] Lambert (1991, p. 341) : "Woman appear to place a higher value than men on social relationships in the workplace, while men place greater importance on career-related job features such as pay, advancement, and autonomy." and Lambert to enumerate studies fuelling this aspect.

technology, work strain and self-esteem carried by the job. The model is built so as not to depart from the most standard analysis of the employment relationship, the exercise undertaken here being more a semantic than syntactical one.

Self-esteem motives are characterized by their dependence on social stereotypes. We rely on this aspect (amply documented[8]) and on the model properties to provide a scenario of selective hiring upon a socio-demographic criterion (above all, upon a gender criterion). Hiring selectivity depends on the characteristics of the job to be filled. We establish, all other things remaining equal, that the characteristics leading to selective hiring in favour of the dominant socio-demographic group (to men in the case of gender disparities) also condition higher compensation: a positive correlation appears between the selectivity of hiring and how well-paid the jobs under consideration are. We thus obtain a vertical segmentation of the labour market where working persons of the dominant group (men) are overrepresented in better-paid jobs.

Our study decomposes into four steps. In the first, we present the basic model and describe its particularities. The second step is devoted to a comparative statics exercise allowing us to make explicit the influence of some particular job characteristics over the outcome of the interaction between the principal and the agent. The third step addresses the issue of hiring selectivity upon a socio-demographic criterion and the fourth, that of the vertical segmentation to which this selectivity drives.

1 The basic model

There are two players: a principal (the employer, *she*) and an agent (the employee, *he*). When she transfers w, the agent exerting effort e, her net surplus is given by $S(e) - w$ where $S(.)$ denotes the gross surplus function associated to the job. We assume that effort is verifiable and that e and w cannot be negative (the agent's liability is limited). The agent, consciously or not, chooses the relation he has to his job. This is modeled as a choice of an identity. There exist two polar identities A and B; the agent can make any conbination of these two. In addition to its gross surplus function, let a job be described by two functions: $\psi(.)$ and $a(.)$. $\psi(e)$ is the work strain experienced by any agent exerting effort e on this job. $a(w, e)$ is the self-esteem (be it intrinsic or resulting from social esteem) he enjoys, assuming he further has identity A and earns w. When an agent has identity B, his self-esteem is taken exogenous set to $b > 0$. The well-being of an agent who has chosen the identity combination $\alpha \in [0, 1]$, exerting effort e and

[8] For some studies raising the issue, see Akerlof and Kranton (2000).

earning w writes

$$u^{\alpha}(w,e) = f(w, \psi(e), x^{\alpha}(w,e))$$

where $x^{\alpha}(w,e)$ combines the amounts of self-esteem $a(w,e)$ and b such that $x^{1}(w,e) = a(w,e)$ and $x^{0}(w,e) = b$.

This writing of the utility function aims at putting the stress on the role, for working persons' well-being, of non wage job characteristics. To this extent, values $\psi(e)$ and $a(w,e)$ must be considered as quasi-objective constituents of well-being at work (that is, exhibiting little variation from one individual to the other): $\psi(.)$ and $a(.)$ chiefly feature a particular job.

One more word as regards what we model as the choice of an identity. It reflects the choice from a worker of a particular *context of self-evaluation*. Considering an agent having identity A (respectively B) comes to consider the case of an individual who views the occupational field (respectively the extraoccupational field) as a context of self-evaluation. It conditions: the motives of an agent's behaviour in general; his motivation within the occupational framework.

1.1 Assumptions

We assume that $f(.)$ and $a(.)$ are twice differentiable in each of their arguments and:[9] $f'_{w} > 0$, $f'_{\psi} < 0$, and $f'_{x} > 0$; $\psi' > 0$; $a'_{w} > 0$, and $a'_{e} > 0$; for all $\alpha \in [0,1]$, $\frac{\partial u^{\alpha}}{\partial e} = f'_{\psi}.\psi' + \alpha.f'_{x}.a'_{e} < 0$. It is further assumed that $D^{2}f$ and $D^{2}a$, respectively the hessian matrices of $f(.)$ and $a(.)$, are such that for all $\alpha \in [0,1]$: $\frac{\partial^{2}u^{\alpha}}{\partial e^{2}} \leq 0$, $\frac{\partial^{2}u^{\alpha}}{\partial w^{2}} \leq 0$ and[10] $\frac{\partial^{2}u^{\alpha}}{\partial e \partial w} \leq 0$. The latter assumption involves in particular: $f''_{w^{2}} \leq 0$, $f''_{\psi w} \leq 0$; ψ'' and $f''_{\psi^{2}}$ such that $f''_{\psi^{2}}.\left(\psi'\right)^{2} + f'_{\psi}.\psi'' \leq 0$. To see full implications of this assumption, see note 1 in the appendix. It should be clear, thus, that we do not depart from the most standard economic assumptions made on the topic but simply propose an arbitrarily detailed description of the content of the employee's utility function.[11] As far function $a(.)$ is considered, its arguments w and e play less as such than by what they mean to an agent having identity A. The very notion of an identity choice involves a willingness of signalling (identity for others) and auto-signalling (identity for oneself). All other things remaining equal, a higher wage whips up the self-esteem of a working person having identity A, $a'_{w} > 0$, as a positive signal (to others in particular) as regards his professional worth; a higher effort whips up his self-esteem, $a'_{e} > 0$, as a positive signal (to himself in particular) as regards

[9] When a property holds for any value of a function's argument, we omit to specify this argument. For instance, $f'_{w} > 0$ stands for $\forall (w, \psi, x) \in \mathbb{R}^{3}_{+}$, $f'_{w}(w, \psi, x) > 0$.

[10] Schwartz's theorem applies.

[11] Of course, the assumptions above are sufficient conditions for equilibrium existence; they are by no means necessary. The conditions to equilibrium existence is not an issue here.

his professional commitment.

What about the way self-esteem amounts attached to polar identities A and B combine to determine $x^{\alpha}(.)$? A *sufficient* condition to reach the result of this paper consists in assuming that, for all $\alpha \in]0,1[$, $x^{\alpha}(.) < \max\{a(.), b\}$ which somewhat comes to favour the idea that self-esteem rather requires specialization *i.e.* the choice of an exclusive context of self-evaluation. Although this assumption makes our reasoning much clearer, it is not a necessary condition.[12] In the remaining of the paper, for the sake of clarity, we will slightly abuse our notations, writing $u^{A}(w,e)$ and $u^{B}(w,e)$ respectively for $u^{1}(w,e)$ and $u^{0}(w,e)$.

1.2 Timing

The timing of the contracting game is as follows. (0) The characteristics of the job, in particular $\psi(.)$ and $a(.)$, are observed both by the principal and the agent. (1) The principal makes a hiring offer consisting in a pair (e,w): she commits to transfering $w \geq 0$ if the actual effort exerted by the agent is higher or equal to e. (2) The agent decides whether to accept the job, adopts an identity (chooses α), and chooses his level of effort. (3) The contract is executed.

Note that the hiring offer cannot be made contingent upon the agent's identity (the latter is assumed not to be verifiable). Given the principal offer (e,w), if the agent *accepts the job*, *chooses the identity combination* α, and *exerts the actual effort* $e_{act} \geq 0$, players' payoffs are given by

$$\begin{aligned} &(S(e_{act}) - w, u^{\alpha}(w, e_{act})) \text{ if } e_{act} \geq e \\ &(S(e_{act}), u^{\alpha}(0, e_{act})) \text{ if } e_{act} < e \end{aligned}$$

while if the agent *refuses the job*, and chooses the combination α, players'payoff are simply $(S_{\emptyset}, u^{\alpha}_{out})$. In the remaining, the focus is on jobs which are actually filed in equilibrium. $S_{\emptyset}$ will thus always be assumed low enough so that the job be not left vacant in equilibrium.

The most important aspect in this modeling of the employment relationship is that the employer has the primary part not only as regards the effort exerted by the agent but also as the subjective relation he has to his job is considered.

1.3 The employer's problem

Clearly, under previous assumptions, $e_{act} > e$ cannot be a dominant strategy, while if $e_{act} < e$ then $e_{act} = 0$. Assuming that she decides to hire the agent, the problem of the principal can thus be written as follows

[12] We will come again to that point in conclusion.

$$\max_{e,w\geq 0} S(e) - w$$

$$\text{s.t.} \begin{cases} \max_\alpha u^\alpha(w,e) \geq \max_\alpha u^\alpha(0,0) & \text{(incentive condition)} \\ \max_\alpha u^\alpha(w,e) \geq \max_\alpha u^\alpha_{out} & \text{(participation condition)} \end{cases}$$

We further assume that: for all $\alpha \in]0,1[$, $u^A_{out} < u^\alpha_{out} < u^B_{out}$; for all $e \in \mathbb{R}_+$, $u^B(0,e) \leq u^B_{out}$. The sine qua non for someone to draw self-esteem from one's job is obviously to have a job. That is what is meant with the first assumption: the more an outsider weights identity A the lower his well-being. The second assumption means that, for someone having identity B, being employed does not provide well-being in itself: it is always better being an outsider than a working-poor. Note that we indeed make a distinction between being an insider without exerting the least effort and being an outsider *i.e.* for an agent having identity A , in particular, between utility amounts $u^A(0,0)$ and u^A_{out}.

Let us define $\underline{u} \equiv \max\left\{u^A(0,0), u^B_{out}\right\}$ and $u(w,e)$, for all $(w,e) \in \mathbb{R}^2_+$, by $u(w,e) \equiv \max\left\{u^A(w,e), u^B(w,e)\right\}$.

Remark 1 *The problem of the principal writes* $\max_{e,w\geq 0} S(e) - w$ *s.t.* $u(w,e) \geq \underline{u}$.

Proof. See the appendix. ■

Incentives and participation conditions only call up polar identities. This directly derives from the assumption that self-esteem achievement requires specialization. For the sake of conveniency, the latter proposition suggests to introduce function $x(.)$ defined for all $(w,e) \in \mathbb{R}^2_+$ by $x(w,e) \equiv \max\{a(w,e), b\}$. Naturally, for all $(w,e) \in \mathbb{R}^2_+$, $u(w,e) = f(w, \psi(e), x(w,e))$.

1.4 Identity dependent indifference curve

Let $\omega^X(.)$, $X \in \{A,B\}$, and $\omega(.)$, be defined, respectively, for all $e \geq 0$, by: $u^X\left(\omega^X(e), e\right) = \underline{u}$ and $u(\omega(e), e) = \underline{u}$. Note that, for all $e \geq 0$, $\omega(e) = \min\left\{\omega^A(e), \omega^B(e)\right\}$. We are obviously dealing with the equation of an agent's indifference curves. These indifference curves are identity dependent and the main purpose in this sub-section is to build a "global" indifference curve, that is: an indifference curve which takes into account the agent's best choice of an identity.

Remark 2 *For* $X \in \{A,B\}$, $\omega^X(0) \geq 0$ *and* $\omega^X(.)$ *is strictly increasing in* e.

Proof. By definition, for all e, $u^X\left(\omega^X(e), e\right) = \underline{u}$ thus, in particular, $u^X\left(\omega^X(0), 0\right) = \underline{u} \geq u^X(0,0)$ which entails $\omega^X(0) \geq 0$. See the appendix for the remaining of the proof. ■

Lemma 3 *For all $e \geq 0$, $\omega^A(e) > \omega^B(e) \Leftrightarrow a\left(\omega^A(e), e\right) < b$.*

Proof. See the appendix. ■

$\omega^X(e)$ represents the lowest amount the employer must transfer to guarantee agent's effort e: the lower the self-esteem experienced by the agent, the higher this amount. The latter lemma means that if the self-esteem induced by some hiring offer is higher for one identity than for the other, the employer transfers less to an agent having the former than to one having the latter. Function $\omega(.)$ is thus strongly related to $a(.)$. To make this relation clear, let us define the function $\tilde{\omega}(.)$, for all e, by $a(\tilde{\omega}(e), e) = b$, and the value $\tilde{e}$ of e by $u(\tilde{\omega}(\tilde{e}), \tilde{e}) = \underline{u}$. Note that $\omega(\tilde{e}) = \omega^A(\tilde{e}) = \omega^B(\tilde{e})$. The curve associated to the function $\tilde{\omega}(.)$ represents some kind of "identity indifference curve" *i.e.* the set of pairs (w, e) such that $A \sim B$. We denote $\tilde{u}_{(0)} = u(\tilde{\omega}(0), 0)$ whatever $X \in \{A, B\}$: this is, for a null effort, the utility of an agent who receives exactly the wage which leaves him indifferent between the two identities. Given $e = 0$: choosing A means that his utility is at least $\tilde{u}_{(0)}$; choosing B, that his utility is at most $\tilde{u}_{(0)}$.

Lemma 4 *If $\underline{u} < \tilde{u}_{(0)}$ then there exists $\tilde{e} > 0$ such that, for all $e \leq \tilde{e}$, $\omega^B(e) \leq \omega^A(e)$ and, for all $e > \tilde{e}$, $\omega^B(e) > \omega^A(e)$. If $\underline{u} \geq \tilde{u}_{(0)}$ then, for all $e \geq 0$, $\omega^B(e) \geq \omega^A(e)$.*

Proof. See the appendix. ■

This lemma states that, for $\underline{u}$ lower than $\tilde{u}_{(0)}$, insofar as e remains below $\tilde{e}$, targetting identity B is less costly to the principal than targetting identity A, whereas for $e > \tilde{e}$, targetting A becomes less costly. This results from the fact that for $e < \tilde{e}$, identity B brings more self-esteem than identity A. Once $\underline{u}$ rises above $\tilde{u}_{(0)}$, identity B never brings as much self-esteem as identity A, so that $\omega^B(e) > \omega^A(e)$ for all e positive.

Remark 5 $\underline{u} = u^A(0,0)\left(\geq u^B_{out}\right) \Rightarrow \underline{u} \geq \tilde{u}_{(0)}$.

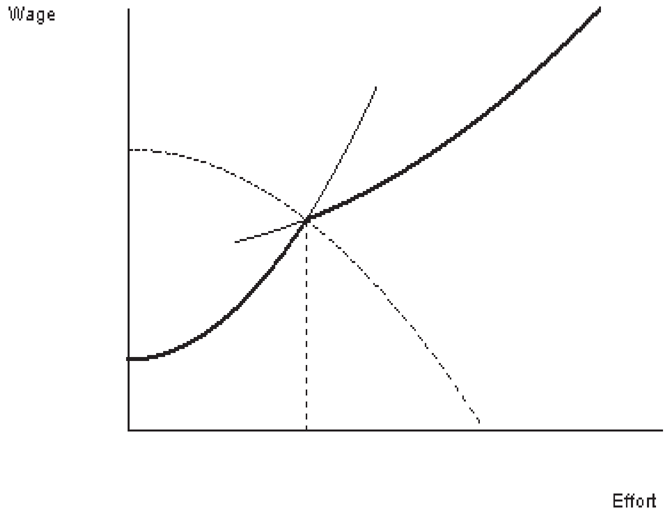
Proof. $u^A(0,0) \geq u^B_{out}$ and $u^B_{out} \geq u^B(0,0) \Rightarrow f(0, \psi(0), a(0,0)) \geq f(0, \psi(0), b)$ and $a(0,0) \geq b$. Yet, by definition of $\tilde{\omega}(.)$, $a(\tilde{\omega}(0), 0) = b$. Therefore: $a(0,0) \geq a(\tilde{\omega}(0), 0)$ and $\tilde{\omega}(0) \leq 0$. As a consequence, $\underline{u} = f(0, \psi(0), a(0,0)) \geq f(\tilde{\omega}(0), \psi(0), a(\tilde{\omega}(0), 0)) \equiv \tilde{u}_{(0)}$. ■

It follows that:

- if $\underline{u} \geq \tilde{u}_{(0)}$ then, for all $e \geq 0$, $\omega(e) = \omega^A(e)$;
- if $\underline{u} < \tilde{u}_{(0)}$ then,

$$\omega(e) = \begin{cases} \omega^B(e) & \text{if } 0 \leq e < \tilde{e} \\ \omega^A(e) & \text{if } e \geq \tilde{e} \end{cases}$$

and thus the indifference curves depicted below.



Identity indifference curve, picturing $\tilde{\omega}(.)$ (dash curve), and indifference curve of an agent depending on his identity, picturing $\omega(.) = \min\left\{\omega^{A}(.), \omega^{B}(.)\right\}$ (bold curve), assuming $\underline{u} < \tilde{u}_{(0)}$.

As one can see, in general, the agent's indifference curves are not fully convex. This obviously impacts on the principal's problem.

Remark 6 $\omega^{A\prime}(\tilde{e}) < \omega^{B\prime}(\tilde{e})$.

Proof. See the appendix. ■

$\omega^{X\prime}$ denotes the marginal rate of substitution[13] of the agent having identity$X$. In $\tilde{e}$, wage variations which leave unchanged the agent's well-being despite unitary variations in effort are of smaller scope if he has identity A than if he has identity B.

It follows that function $\omega(.)$ is differentiable on $\mathbb{R}_{+} - \{\tilde{e}\}$; it is not differentiable in $\tilde{e}$. The way we introduce self-esteem motives in the analysis of the employment relationship has a twofold impact thus. First, it entails non-convex indifference curves; second, it entails not-everywhere differentiable $\omega(.)$. We deal with the consequences of these aspects in the next sub-section.

1.5 Optimal hiring offer

$S(.)$ is defined on $\mathbb{R}_{+}$ and twice differentiable on $\mathbb{R}_{+}$. It is assumed that: for all $e \in \mathbb{R}_{+}$, $S'(e) > 0$ and $\lim_{e \to 0} S'(e) = +\infty$; for all $e \in \mathbb{R}_{+}$, $S''(e) < 0$. Let (w^{*}, e^{*}) denotes the employer's optimal hiring offer.

[13] $\omega^{X\prime} = \left.\frac{dw}{de}\right|_{du^{X}=0}$.

Remark 7 *$u(w^*, e^*) = \underline{u}$ i.e. the constraint is binding in equilibrium.*

Proof. See the appendix. ■

The fact that the constraint is binding in equilibrium rewrites $w^* = \omega(e^*)$.

Lemma 8 *$(w^*, e^*) \neq (\tilde{w}, \tilde{e})$ (or, equivalently, $a(w^*, e^*) \neq b$).*

Proof. The proof amounts to show that, starting from $(\tilde{w}, \tilde{e})$, one can always find profitable deviation (see the detail of the proof in the appendix). ■

This means that the contract offered by the employer induces the worker to have a pure identity in equilibrium: A if $a(w^*, e^*) > b$, B if $a(w^*, e^*) < b$. It further follows that $\omega(.)$ is differentiable in $e = e^*$. We can thus rely on standard optimality conditions.

Remark 9 *An optimal hiring offer (w^*, e^*) exists; it satisfies $S'(e^*) = \omega'(e^*)$ and $w^* = \omega(e^*)$.*

Proof. The hypotheses of the model ensure an interior solution (see the details in appendix). ■

To the extent that non-convexity of indifference curves is local, it does not compromise the existence of an optimal hiring offer; nor does it lead to degenerated cases. It follows that this optimal hiring offer is ruled by the most standard microeconomic trade-offs which we will not analyze here in greater details. Let us rather focus on identity targeted by the contract favoured by the employer.

1.6 Equilibrium

In equilibrium, identity adopted by the agent is fully determined by the contract offered by the principal, which itself derives from profit maximization. Let us define $\left(w^X, e^X\right)$, $X \in \{A, B\}$ by $S'\left(e^X\right) = \omega^{X\prime}\left(e^X\right)$ and $w^X = \omega^X\left(e^X\right)$.

Remark 10 *If $\underline{u} \geq \tilde{u}_{(0)}$ then $(w^*, e^*) = \left(w^A, e^A\right)$.*

Proof. See the appendix. ■

This first case could be regarded as the standard one. Remember indeed that, by lemma 4, for $\underline{u} \geq \tilde{u}_{(0)}$, the lowest amount the principal must transfer to ensure some given effort is always higher if the agent has identity B than if he has identity A. It is as if there was only one available identity, identity A; since our background assumptions do not depart from the standard ones, we are brought back to the standard model.

Remark 11 *Suppose* $\underline{u} < \tilde{u}_{(0)}$.
If $S'(\tilde{e}) \geq \omega^{B\prime}(\tilde{e})$ *then* $(w^*, e^*) = (w^A, e^A)$.
If $S'(\tilde{e}) \leq \omega^{A\prime}(\tilde{e})$ *then* $(w^*, e^*) = (w^B, e^B)$.

Proof. See the appendix. ■

Here, the assumptions specific to our model matter ($\underline{u}$ is small enough) and we focus on cases where conclusions are the most direct. Condition $S'(\tilde{e}) \geq \omega^{B\prime}(\tilde{e})$ entails that the effort the principal would demand from an agent having identity B is higher than $\tilde{e}$ ($e^B > \tilde{e}$) and requires a transfer which is strictly higher than what an agent with identity A would ask for, all other things remaining equal. The principal's interest is obviously to target identity A. Condition $S'(\tilde{e}) \leq \omega^{A\prime}(\tilde{e})$ drives just to the converse conclusion.

All other cases between, that is for $\omega^{B\prime}(\tilde{e}) > S'(\tilde{e}) > \omega^{A\prime}(\tilde{e})$, one can hardly states simple conditions on job characteristics and/or $f(.)$ such that, in equilibrium, one identity overrides the other. We can, however, draw simple general conclusions as regards the differences between the two candidates.

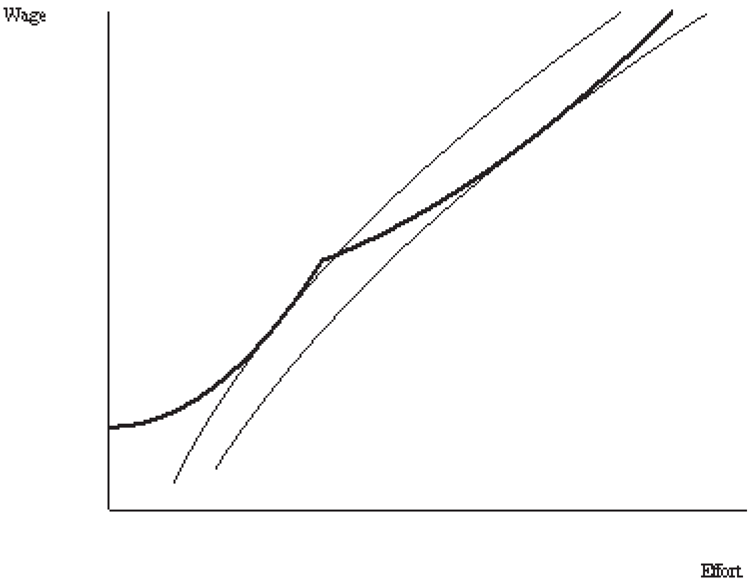
Remark 12 *If* $\underline{u} < \tilde{u}_{(0)}$ *and* $\omega^{B\prime}(\tilde{e}) > S'(\tilde{e}) > \omega^{A\prime}(\tilde{e})$ *then equilibrium candidates* (w^A, e^A) *and* (w^B, e^B) *are such that* $w^A > w^B$ *and* $e^A > e^B$.

Proof. This result directly derives from the optimality condition put forward in proposition 9 (see the detail of the proof in the appendix). ■

Things take place as if there were two qualities of workers except that these two qualities have got to do with a unique individual.[14] The quality of provided work results from the contract designed by the principal. She can either choose a contract which induces a self-esteem motivation from the agent or not. This choice depends on the *job characteristics*: if these characteristics provide little intrinsic rewards, the only source of motivation is wage, motivation is purely extrinsic; if these characteristics are propitious, an intrinsic work motivation appears.

We now provide a graphical analysis of situations where $\omega^{B\prime}(\tilde{e}) > S'(\tilde{e}) > \omega^{A\prime}(\tilde{e})$. Let us first consider the case where the contract chosen by the employer is such that $A \succ B$: $\pi^A = S(e^A) - \omega^A(e^A) > \pi^B = S(e^B) - \omega^B(e^B)$. The following graph extends the previous one by ploting isoprofit curves depending on employer's contract choice targeting one identity or the other (B for the lower convex part of the indifference curve, A for the upper convex part).

[14] This suggests another interpretation of the model with a labour pool made up of two qualities of workers and an employer designing an employment offer targetting one category over the other. Although this interpretation avoids dealing with psychology, it misses our point as regards the endogeneity of work motivation which we view as particularly relevant as far male-female disparities in the labour market are considered.



In equilibrium, the contract chosen by the principal targets identity A.

To develop the analysis, it is convenient to rewrite the condition $\pi^B < \pi^A$ as[15]

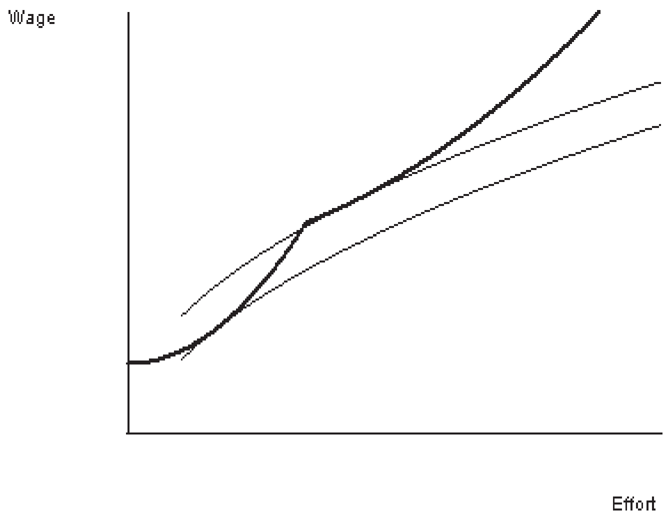
$$\int_{e^B}^{\tilde{e}} \left(\omega^{B\prime}(e) - S'(e)\right) de < \int_{\tilde{e}}^{e^A} \left(S'(e) - \omega^{A\prime}(e)\right) de$$

and to consider the choice of the employer starting from $(\tilde{w}, \tilde{e})$. The gain for the employer when choosing a contract which targets identity B is in wage reduction, while the cost is in lost gross surplus (since, from $\tilde{e}$, the effort required must be reduced). $\omega^{B\prime}(e)$ is the reduction in labour cost resulting from a slight decrease in the effort required on the job; $-S'(e)$ represents corresponding loss in gross surplus. Conversely, targeting identity A allows a gain in gross surplus $S'(e)$ for an increase in labour cost $-\omega^{A\prime}(e)$. The latter condition claims that identity A is preferred when the gain in net surplus associated to the shift from $\tilde{e}$ to $e^A > \tilde{e}$ exceeds the gain associated to the sliding from $\tilde{e}$ to $e^B < \tilde{e}$.

We now consider the case where the contract chosen by the employer is such that $A \prec B$.

[15] Since $\omega^A(\tilde{e}) = \omega^B(\tilde{e})$, $S(e^B) - \omega^B(e^B) < S(e^A) - \omega^A(e^A)$ is equivalent to

$$\begin{aligned}
\omega^B(\tilde{e}) - S(\tilde{e}) + S\left(e^B\right) - \omega^B\left(e^B\right) &< S\left(e^A\right) - \omega^A\left(e^A\right) + \omega^A(\tilde{e}) - S(\tilde{e}) \\
\omega^B(\tilde{e}) - S(\tilde{e}) - \left(\omega^B\left(e^B\right) - S\left(e^B\right)\right) &< S\left(e^A\right) - \omega^A\left(e^A\right) - \left(S(\tilde{e}) - \omega^A(\tilde{e})\right) \\
\int_{e^B}^{\tilde{e}} \left(\omega^{B\prime}(e) - S'(e)\right) de &< \int_{\tilde{e}}^{e^A} \left(S'(e) - \omega^{A\prime}(e)\right) de
\end{aligned}$$



In equilibrium,the contract chosen by the principal targets the identity B.

As one can see, technology has a critical part in the determining of the targeted identity: from the graph depicting an equilibrium leading to $A \succ B$ to this latter one, the only change is a reduction in the elasticity of production to effort (all other things kept constant).

As we said already, it proves delicate to obtain general and simple conditions for the emerging of an equilibrium exhibiting such or such polarity (A or B). We provide an analysis of the impact of key parameters on this polarity in the next section.

2 Jobs characteristics and the attraction of identities A and B

All other things remaining equal, how marginal changes in the characteristics of a job impact the relative attraction of identities A and B? Here is the question raised in the comparative statics analysis below. Our interest also takes in the impact of these changes on the transfer and effort amount stipulated by optimal contracts. Our starting point is a situation where $\underline{u} < \tilde{u}_{(0)}$ and thus $u^A(0,0) < u^B_{out}$.[16] Let δz denotes a variation in some quantity z. When z is a function of some variable r, we denote $\delta_r z(r)$ a variation of z *for r kept constant*. The analysis below relies on the following definition.

Definition 13 *A change δz in some parameter z will be said to increase the attraction of identity A (respectively B) if, all other things remaining equal, $\delta z > 0 \Rightarrow \delta\pi^A > \delta\pi^B$ (respectively, $\delta\pi^B > \delta\pi^A$).*

[16] See remark 5.

2.1 The role of technology

Here, we consider a shock on the marginal return on effort defined, for all e, by $\delta_e S'(e) = \delta S'$.

Proposition 14 *For* $X \in \{A, B\}$, $\frac{\delta e^X}{\delta S'} > 0$ *and* $\frac{\delta w^X}{\delta S'} > 0$.

Proof. See the appendix. ■

Whatever the identity under consideration, a positive shock on the marginal return on effort induces an increase in the wage and effort required by the contract.

Proposition 15 $\delta S' > 0$ *increases the attraction of identity* A.

Proof. See the appendix. ■

Here is confirmed what the graphical analysis in previous section suggested: the higher the marginal return on effort, the more attractive identity A to the principal. The reason is obviously that the rise in effort requirements (whatever the targeted identity) provides self-esteem to an agent having identity A which moderates the increase in wage needed to meet the incentive condition. Note that a change in technology which would not affect the marginal return on effort would leave the choice of the employer unchanged.

2.2 The role of work strain

Let's consider a variation $\delta\psi$ such that, for all e, $\delta_e\psi(e) = \delta\psi > 0$.

Proposition 16 *For* $X \in \{A, B\}$:
(1) if marginal rates of substitution of effort for wage are non decreasing in work strain then, all other things remaining equal, $\delta\psi > 0 \Rightarrow \delta e^X \leq 0$;
(2) if income effects dominate substitution effects then, all other things remaining equal, $\delta\psi > 0 \Rightarrow \delta w^X > 0$.

Proof. See the appendix. ■

Under previous assumptions, all other things remaining equal, $\delta\psi > 0$ affects the *material* well-being of the agent to the same extent has he identity A or B; but, since $\delta w^X > 0$, this negative impact is better compensated if he has identity A since he benefits a gain in self-esteem attached to the holding of a more rewarding job. A reduction in effort requirement $\left(\delta e^X \leq 0\right)$ improves the *material* well-being of the agent to the same extent has he identity A or B, but having the former, he suffers a loss in self-esteem from holding a less challenging job. All other things remaining equal, the impact of a change in work strain, thus, is non trivial and depends on the relative weight of each of the previous arguments.

In the statement below, we use the following short writings: u^X for $u^X\left(w^X, e^X\right)$, f^X for $f\left(w^X, \psi\left(e^X\right), x\left(w^X, e^X\right)\right)$, a^X for $a\left(w^X, e^X\right)$, ω^X for $\omega^X\left(e^X\right)$ with $X \in \{A, B\}$.

Remark 17 *$\delta\psi > 0$ increases identity A attraction if and only if $-\frac{f_\psi^{B\prime}}{u_w^{B\prime}} > -\frac{f_\psi^{A\prime}}{u_w^{A\prime}}$ (> 0).*

Proof. See the appendix. ■

$-\frac{f_\psi^{X\prime}}{u_w^{X\prime}}$ represents the marginal rate of substitution of wage for rest. The latter remark thus simply states that, for $\delta\psi > 0$, the attraction of identity A will rise if and only if the increase in wage necessary to offset $\delta\psi$ is lower has the agent identity A than B. What mechanisms fuel the latter observation? To answer this question, let us consider a case ensuring that the previous inequality holds. Suppose $(0 <) \, u_w^{B\prime} = f_w^{B\prime} < f_w^{A\prime} + f_x^{A\prime} a_w^{A\prime} = u_w^{A\prime}$ and $(0 >) \, f_\psi^{A\prime} > f_\psi^{B\prime}$. In the initial situation, the agent is more sensitive to wage variations has he identity A than B. This is because self-esteem motives associated to wage (as represented by the term $f_x^{A\prime} a_w^{A\prime} > 0$) offset its decreasing marginal utility. Furthermore, work strain marginal disutility $(-f_\psi^{\prime})$ is such that the agent is more sensitive to work strain variations has he identity B than A. The agent finds a work strain increase $(\delta\psi > 0)$ less painfull with identity A than with identity B while correlative wage increase is more enjoyable in the former than in the latter case. The attraction of identity A rises.

A variant on our model deserves particular attention. Suppose the self-esteem of an agent with identity A actually writes $a(w, e) = \check{a}(w, \psi(e))$, all other things remaining unchanged, with $\check{a}_\psi^{\prime} > 0$. Under such behavioral assumption, this is the strain experienced at work rather than effort in itself which conveys positive self-signal of work commitment. We stick to the assumption that $\frac{\delta_{(w,e)} u^A}{\delta\psi} < 0$ which requires $f_\psi^{\prime} + f_x^{\prime} \check{a}_\psi^{\prime} < 0$.

Proposition 18 *Suppose $\psi'' \geq 0$ and, for all $(w, e) \in \mathbb{R}_+^2$, $a(w, e) = \check{a}(w, \psi(e))$. Then $\delta\psi > 0$ increases identity A attraction.*

Proof. See the appendix. ■

For a fairly natural behavioral assumption thus, the higher work strain, the more attractive the identity A to the principal. Note that this assumption plays as a sufficient condition to get a positive relation between work strain and the attraction of identity A: previous steps bring out that the class of situations leading to such a conclusion is much larger.

Comparative statics enables us to have clearer ideas of the relative attraction of each identity, depending on job characteristics. All other things remaining equal, $\delta S'$ or $\delta\psi$ may obviously represent not just a marginal change in a given job characteristics but some aspects differentiating two jobs similar in any other features. The latter analysis can thus be used to distinguish between the jobs the characteristics of which rather call for identity A from the other jobs.

Let us make a few comments as regards background correlation between identity A attraction for some given job and the amount it pays. All other things remaining equal, jobs which exhibit the highest marginal return on effort are: the better paid *and* characterized by a stronger identity A attraction. When conditions on self-esteem motives put forward in the latter subsection are met, jobs exhibiting the highest work strain are: the better paid *and* characterized by a stronger identity A attraction. Hence, the model suggests *a positive correlation between the amount a given job pays and the employer's predilection for identity A.* This correlation plays a crucial part in our interpretation of male-female vertical occupational segregation.

3 Selective hiring

Let us come to the applied phase of the analysis and tackle the problem of sociodemographic disparities in the labour market. In this section, we use the model presented above to propose an explanatory scenario of selective hiring on apparently non-productive criterions (such as age, gender, race...). Let us assume some employer (the principal of previous sections) faces a population made up of two types of workers, $\theta \in \{0, 1\}$, distinguished by their "ability" (or ease) to draw self-esteem in or outside work.[17] To stand by the male-female example, type 0 refers to women whereas type 1 refers to men. For given job characteristics, the amount of self-esteem experienced by an agent of type θ now writes $x_\theta(w, e)$. Our questioning here is whether the employer will favour one type over the other depending on the characteristics of the job to be filled (selective hiring). If the employer proves indifferent between types, we will say that hiring is non-selective. Let us stress again that in our model, a job is characterized by: its technology (for a given effort, is marginal productivity high or low?), its strain (is work strain high or low?), and the self-esteem one can derive from it; that is, by the triple $(S(.), \psi(.), a(.))$. The role of these characteristics, however, will remain implicit below. Indeed, for the sake of clarity, we directly deal with net surplus which sum up job characteristics.

Two scenarios could be considered here: the case where $b_0 > b_1$; and the case where, for all $w, e \geq 0$, $a_0(w, e) < a_1(w, e)$. The first of those scenarios corresponds to the assumption that,

[17] The argument introduced here is completely independent from market mechanisms. In what follows, we thus put these mechanisms aside, which comes formally to assume that two workers at least (one of each type) apply to a given hiring offer.

for some reason, agents of type 0 have a greater "ability" to enjoy self-esteem *outside work*, all other things remaining equal, than agents of type 1. To refer to the example of male/female differences, let's say that because of a socially appreciated maternal role, female have greater opportunities than male to enjoy self-esteem outside work *i.e.* that outside-work is a context of self-evaluation more favourable to women than to men. The second case corresponds to the assumption that, for some reason, agents of type 1 have greater "ability" to enjoy self-esteem *inside work* than agents of type 0. To refer to the example of male-female differences, let's say that, all other things remaining equal, because manliness is socially appreciated at work, men enjoy greater self-esteem than women within work field *i.e.* work is a context of self-evaluation more favourable to men than to women. For conciseness, we only consider the second of these scenarios. Note, however, that the two trails are equivalent to a large extent and that, in both scenarios, the difference between type 0 and type 1 workers can be taken as small as desired, provided it exists.[18]

Let us assume that, for all $w, e \geq 0$, $a_1(w,e) - a_0(w,e) = \delta a > 0$ ($\delta a \to 0$ possibly). The question is whether this will induce selective hiring in favour of male or female, depending on the characteristics of the job under consideration.

Proposition 19 *Suppose* $0 < \delta a \leq \frac{u^B_{out} - u^A_0(0,0)}{f'_x(0,\psi(0),a_0(0,0))}$. *Then, hiring is selective and favourable to job applicants of type* 1 *if and only if*

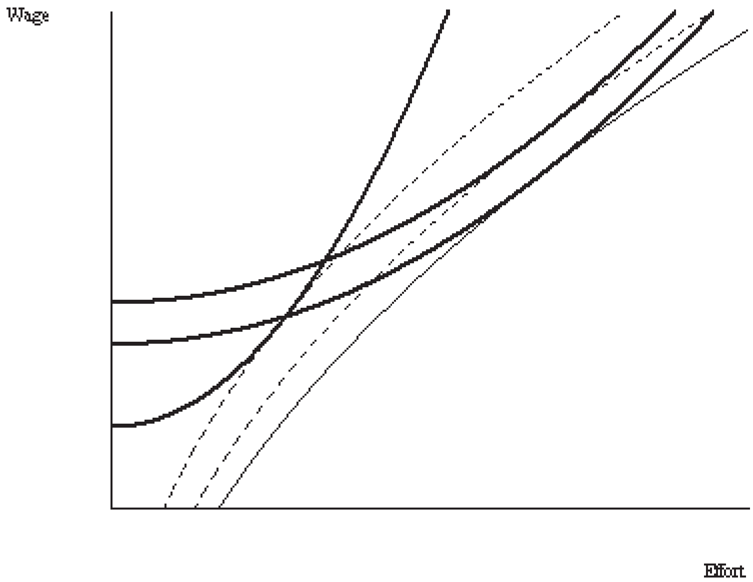
$$\pi^B_0 - \pi^A_0 < \frac{f^{A_0\prime}_x}{u^{A_0\prime}_w}\delta a \qquad (C)$$

where u^{A_0} *is a short writing for* $u\left(w^A_0, \psi\left(e^A_0\right), a\left(w^A_0, e^A_0\right)\right)$.

Proof. The proof consists in establishing: that $\pi^B_1 = \pi^B_0$ on the one hand; condition C which is equivalent to $\pi^A_1 > \pi^A_0$ on the other (see the detail of the proof in the appendix). ■

Here follow the graphs picturing successively the case with and without selective hiring.

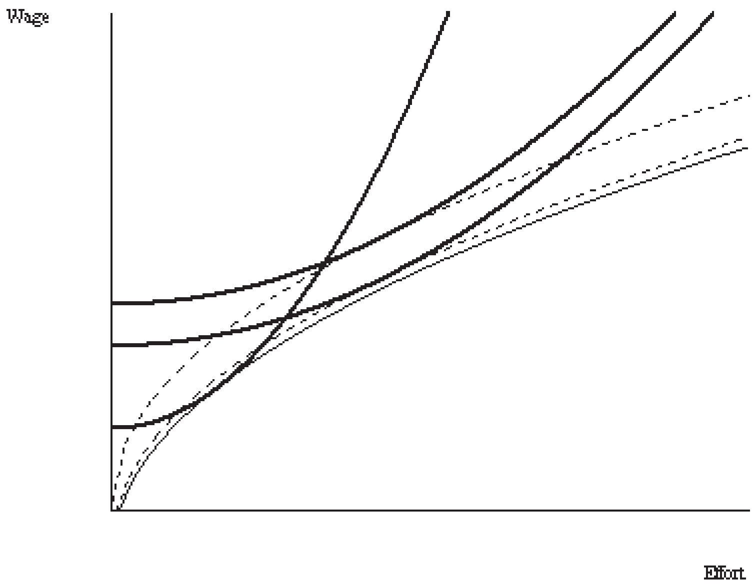
[18] The analysis of the case $b_0 > b_1$ is available from the author.



The case of a job the characteristics of which are such that $\delta a > 0$ leads to selective hiring.

Premises entails that $\underline{u}$ is the same for both types of workers. Then, under the necessary and sufficient condition C, *which holds in particular for* $\pi_0^A > \pi_0^B$, type 1 applicants are favoured over type 0 by the employer. The reason is simply that the contract designed by the employer, whatever the worker's type, relies on a self-esteem motivation which is presumed stronger for type 1 workers.

If the condition C does not hold (as in the graph below), the contract designed by the employer is unaffected by $\delta a > 0$ to the extent that it actually targets identity B for which types 0 and 1 workers have a similar (exclusively extrinsic) motivation.



The case of a job the characteristics of which are such that $a_0(.) < a_1(.)$ does not lead to selective hiring.

Note that, for $u_{out}^{B} > u_0^{A}(0,0)$ one can always find δa small enough so that the previous condition could be applied.[19]

In this section, we have characterized configurations leading to selective hiring. These configurations resulted from the characteristics of the job under consideration. As far as male-female differences in professional achievement are considered, not only are some jobs reserved to men, but these jobs, all other things remaining equal, prove better paid than non-selective ones. We obviously get on to the core of our argument. We now connect conclusions drawn in this section to the observations made earlier as regards the role of technology and work strain in the decision of the employer to design contract targeting identity A or B.

4 Vertical occupational segregation

Let us consider a labour market displaying jobs with different non wage characteristics. For a given level of effort, differences may concern in particular: how much these jobs yield to the firm, the strain endured by their holders, the self-esteem one can draw from them. Relying on previous steps, our purpose in this section is to provide a qualitative analysis of how this differentiation may condition socio-demographic disparities in the labour market and, above all, male-female disparities.

4.1 Occupational segregation

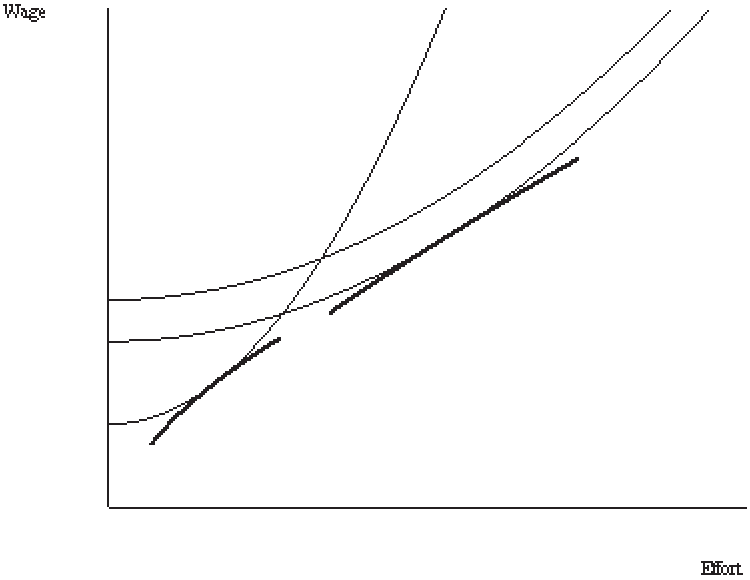
As we did in the last section, we focus on the hypothesis that type 1 workers (male) have an easier access to self-esteem through work than those of type 0 (female). If some job characteristics are such that the optimal contract targets the identity B regardless of employee's type (0 or 1), there is no need for the employer to hire selectively upon socio-demograhic criterion (in particular, upon gender). In other words, jobs which do not require some kind of intrinsic motivation should not give rise to sociodemographic (gender) selective hiring. Conversely, if some job characteristics are such that the optimal contract targets the identity A, this will first be the case for type 1 workers (male): hiring will be selective at the expense of type 0's. In other words, jobs the characteristics of which let the employer expect an intrinsic motivation should be held by type 1 workers. Overall, one should observe an overrepresentation of type 1 workers in jobs inducing an intrinsic work motivation and a related overrepresentation of type 0 workers in other jobs.

[19] The analysis of the case $u_{out}^{B} \leq u_0^{A}(0,0)$ is available upon request to the author.

Let's be more precise: which job characteristics induce the employer to hire selectively? Within the framework of our model this question restates: which job characteristics make it profitable for the employer to induce the identity A? The analysis of section 2 suggests some answers.

Remark 20 *A job is all the more likely to be selective (to require identity A) that, all other things remaining equal, it exhibits a higher marginal return on effort.*

To illustrate this point, let's consider two classes of jobs, j and J, only differentiated from each other by the elasticity of production to effort. The picture below illustrates the reason why class J (exhibiting a higher marginal return on effort) is more likely to give rise to selective hiring than class j.



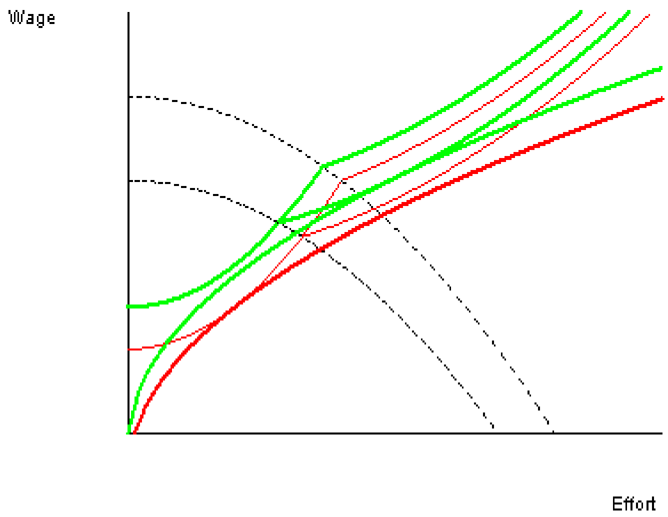
Two jobs j and J with different marginal returns to effort: $S_j' < S_J'$ all other things remaining equal.

Because jobs of class J exhibit a higher return to effort, it proves profitable for the employer to induce identity A in which case hiring is selective.

Remark 21 *A job is all the more likely to be selective (to require identity A) that, all other things remaining equal, its holding induces a higher work strain.*

In the picture below, job classes j and J are only differentiated by the strain their holding induces: for all $e \geq 0$, $\psi_J(e) - \psi_j(e) = \delta\psi > 0$, all other things remaining equal.[20] It illustrates how jobs inducing higher work strain may give rise to selective hiring.

[20]In particular, for all $e \geq 0$, $S_j(e) = S_J(e)$.



Two jobs j and J with different work strains, $\psi_J > \psi_j$, all other things remaining equal.

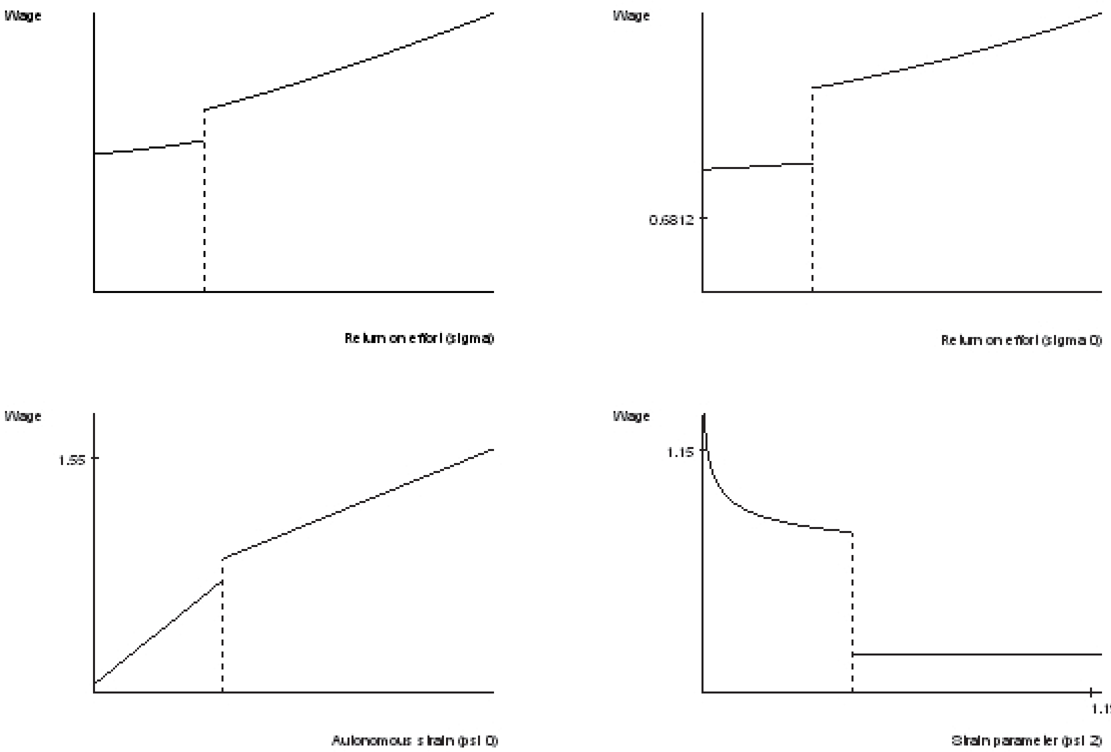
Because holding a job of class J induces higher work strain, it proves profitable for the employer to induce the identity A in which case hiring is selective.

From the perspective of the employer, workers' type makes little difference as regards class j jobs. On the contrary, class J jobs give rise to a selective hiring in the favour of type 1 workers. *One should thus observe an overrepresentation of type* 1 *workers in the jobs of class* J *and a related overrepresentation of type* 0 *workers in the jobs of class* j*.*

4.2 Sociodemographic gaps in average earnings

Why could the occupational segmentation obtained above be described as vertical? Because jobs of classes j and J are bound not to lead to similar average earnings. Average earnings should be higher among workers holding a job in class J. The first reason for this is that jobs of class J pay an efficiency wage corresponding to a relatively higher effort requirement. The second reason is that the characteristics which makes a particular job belonging to the class J, as the latter two pictures show, also condition higher pays. In the first case, that is because jobs J have a higher marginal return on effort; in the second, because their holding leads to a higher work strain.

Our model thus backs a positive correlation between wage and the propensity of employers to target identity A. The next graphs illustrate this correlation within the framework of a quasi-linear specification of the model, assuming in particular: $\psi(e) = \psi_2 e^2 + \psi_0$ and $S(e) = 2\sigma e^{\frac{1}{2}} + \sigma_0 e$ (see the full specification of the model in the appendix). Our interest goes on the way some change in non wage characteristics of jobs influence on the amount they pay *i.e.*, all other things remaining equal, on the role of parameters ψ_0 and ψ_2 (characterizing work strain attached to a job) on the one hand, the role of parameters σ and σ_0 (characterizing the technology) on the other hand. Changes in ψ_0 and σ_0 correspond to those we analysed in section 2.



Whatever the productivity parameter considered, the shift from identity B to identity A arises for increasing wage values. This is also the case as regards parameters reflecting work strain. Graphs above further illustrate the discontinuity of the relation between non wage characteristics of a job and the amount it pays. This discontinuity obviously corresponds to the coincidence of the shift in identity with a jump in the effort required, all other things remaining equal.

5 Conclusion

The analysis presented above suggests men's jobs are better paid in average than women's jobs because: they pay an efficiency wage whereas women's do not; they tend to be more demanding and/or more strenuous. The explanation for male-female gap in earnings should thus be found in non wage characteristics of jobs hold by each gender. This interpretation is far from new and many evidence exist which support it. We can for instance come again on the OECD report mentioned in introduction which summons up studies suggesting an under-utilization of women skills at work. Inspite of educational attainment levels that are similar for women and men or even in favour of women, white-collar women engage in writing and reading at work less frequently and/or with less variety than white-collar men in all OECD countries examined; fewer women than men declare that they are carrying out complex tasks in their jobs; more women than men feel that the demands imposed on them by their jobs are too low relative to their skills, conversely, fewer women than men think they are too high. Overall, it seems that men's jobs are indeed more demanding than women's.[21] Lambert (1991) finds matching evidence from a different perspective. The initial question of her study is why men and women maintain comparable levels of job satisfaction even though women's jobs are less gratifying in terms of both intrinsic (related to non wage job characteristics) and extrinsic (pecuniary) rewards. She provides an explanation by taking into account the social rewards and the stress attached to individuals' working experiences. Although women's jobs are on the average less rewarding, they are also on the average less stressful and appear to provide greater social rewards: these combined aspects prove strong enough to offset the lack of intrinsic rewards provided by women's jobs. Men are more likely to be both inundated with, and more sensitive to, conflicting and overwhelming job tasks, resulting in greater psychological involvement in work at the cost of reduced job satisfaction.

Let us come to the question raised in introduction: how and why does the labour market sort women and men into jobs with different characteristics and productivities? Our answer relies on the assumption of a male-female asymmetry as regards the ease with which each gender draws self-esteem from work. This assumption results in diverging strategies of self-esteem achievement which, in turn, fuel labour market segmentation. The model explains how men can be over-represented in jobs soliciting an intrinsic motivation; women in jobs which do not. It is indeed employer's best interest to mobilize an intrinsic motivation if, all other things remaining equal, the return to effort is high: men are overrepresented in the most productive jobs. Similarly, all other things remaining equal, the higher the work strain, the higher the required compensation and the more prone the employer to mobilize an intrinsic motivation: men are overrepresented among the most strenuous jobs.

As far normative aspects of the model are considered, a point deserves particular attention to avoid misinterpretations. One could erroneously think that matters of equal opportunities are wiped out from the analysis: women have preferences which drive them holding jobs with little intrinsic rewards. That is not the case! One should keep in mind that, in the model,

[21] See *OCDE employment outlook* (2002, p.93).

gender differences in preferences are *ex post* differences. It directly results from the selective hiring practices of some employers which, in turn, are a consequence of gender differences in self-esteem achievement oppotunities inside or outside work. This state of things is due to social stereotypes. Our model, therefore, supports the need to fight social stereotypes not only as a fundamental means to achieve equal opportunities between men and women but also as a way to make a more efficient use of female human ressources.

Several assumptions of the model deserves complementary comments: to what extent could they be relaxed without compromising our result? It should be clear, at this stage of the discussion, that the assumption $x^{\alpha}(.) < \max\{a(.), b\}$ is far from being necessary to our result. Yet, having a precise idea of the range of psychological hypotheses consistent with our conclusions would obviously be desirable. Another investigation necessary to properly assess the scope of our argument would be to study its interplay with (labour) market mechanisms (in the spirit of Arrows assessing Becker's model of discriminatory tastes). This is the next step on our agenda.

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[10] OCDE (2002), *OECD Employment Outlook*, Paris, OECD Publication.

6 Annexe

Note 1 Given the assumptions stated concerning the first order derivatives of $f(.)$ and $a(.)$, any hessian matrices $D^2 f$ and $D^2 a$ satisfying, for all $\alpha \in [0,1]$,

$$\begin{aligned}
\frac{\partial^2 u^\alpha}{\partial w^2} &= f''_{w^2} + \alpha\left(\alpha f''_{x^2}\left(a'_w\right)^2 + 2 f''_{wx} a'_w + f'_x a''_{w^2}\right) \leq 0 \\
\frac{\partial^2 u^\alpha}{\partial e^2} &= f''_{\psi^2}\left(\psi'\right)^2 + f'_\psi \psi'' + \alpha\left(2 f''_{\psi x}\psi' a'_e + \alpha f''_{x^2}\left(a'_e\right)^2 + f'_x a''_{e^2}\right) \leq 0 \\
\frac{\partial^2 u^\alpha}{\partial e \partial w} &= f''_{\psi w}\psi' + \alpha\left(f''_{\psi x}\psi' a'_w + f''_{xw} a'_e + \alpha f''_{x^2} a'_w a'_e + f'_x a''_{ew}\right) \leq 0
\end{aligned}$$

meet the requisits of the model.

The principal's problem writes

$$\begin{aligned}
&\max_{e,w\geq 0} S(e) - w \\
&\text{s.t.} \begin{cases} \max_\alpha u^\alpha(w,e) \geq \max_\alpha u^\alpha(0,0) \\ \max_\alpha u^\alpha(w,e) \geq \max_\alpha u^\alpha_{out} \end{cases}
\end{aligned}$$

Remark 1 *The problem of the principal writes* $\max_{e,w\geq 0} S(e) - w$ *s.t.* $u(w,e) \geq \underline{u}$.

Proof. We show first that $\max_\alpha u^\alpha(w,e) = u(w,e)$. Let us consider $\alpha \in \left]0,1\right[$. $\forall (w,e) \in \mathbb{R}^2_+$ such that $a(w,e) < b$, $\max\left\{u^A(w,e), u^\alpha(w,e)\right\} < u^B(w,e)$; $\forall (w,e) \in \mathbb{R}^2_+$ such that $a(w,e) = b$, $u^A(w,e) = u^B(w,e) > u^\alpha(w,e)$; $\forall (w,e) \in \mathbb{R}^2_+$ such that $a(w,e) > b$, $u^A(w,e) > \max\left\{u^\alpha(w,e), u^B(w,e)\right\}$. Thus, $\forall (w,e) \in \mathbb{R}^2_+$, $\max_{\alpha\in[0,1]} u^\alpha(w,e) = \max\left\{u^A(w,e), u^B(w,e)\right\} = u(w,e)$. Let us prove, now, that $\max\left\{\max_\alpha u^\alpha(0,0), \max_\alpha u^\alpha_{out}\right\} = \underline{u}$. Our assumptions directly lead to $\max_\alpha u^\alpha_{out} = u^B_{out}$ while $\max_\alpha u^\alpha(0,0) = \max\left\{u^A(0,0), u^B(0,0)\right\}$ derives from previous step of the proof. Consequently the participation condition writes $u(w,e) \geq \max\left\{\max\left\{u^A(0,0), u^B(0,0)\right\}, u^B_{out}\right\}$ with $\max\left\{\max\left\{u^A(0,0), u^B(0,0)\right\}, u^B_{out}\right\} = \max\left\{u^A(0,0), u^B(0,0), u^B_{out}\right\}$ and, since by assumption $u^B(0,0) \leq u^B_{out}$, $\max\left\{u^A(0,0), u^B(0,0), u^B_{out}\right\} = \max\left\{u^A(0,0), u^B_{out}\right\} = \underline{u}$. ∎

Remark 2 *For $X \in \{A, B\}$, $\omega^X(0) \geq 0$ and $\omega^X(.)$ is strictly increasing in e.*

Proof. This is standard: for $X \in \{A, B\}$,

$$u^X\left(\omega^X(e), e\right) = \underline{u} \Rightarrow \frac{d\omega^X}{de}(e) = -\frac{\frac{\partial u^X}{\partial e}\left(\omega^X(e), e\right)}{\frac{\partial u^X}{\partial w}\left(\omega^X(e), e\right)}$$

$\frac{\partial u^X}{\partial e} < 0$ and $\frac{\partial u^X}{\partial w} > 0$ entails $\frac{d\omega^X}{de} > 0$, hence the conclusion. ■

Let us recall for the remaining, that $\underline{u} = \max\left\{u^A(0,0), u^B_{out}\right\} \geq u^B(0,0)$ and that f'_w, f'_x, a'_w and a'_e are all strictly positive.

Lemma 3 *For all $e \geq 0$, $\omega^A(e) > \omega^B(e) \Leftrightarrow a\left(\omega^A(e), e\right) < b$.*

Proof. For all $e \geq 0$, $\omega^A(e) > \omega^B(e) \Rightarrow a\left(\omega^A(e), e\right) < b$. Indeed, $\omega^A(e) > \omega^B(e)$ implies $f\left(\omega^B(e), \psi(e), b\right) < f\left(\omega^A(e), \psi(e), b\right)$ but $f\left(\omega^B(e), \psi(e), b\right) = \underline{u} = f\left(\omega^A(e), \psi(e), a\left(\omega^A(e), e\right)\right)$ so that $f\left(\omega^A(e), \psi(e), a\left(\omega^A(e), e\right)\right) < f\left(\omega^A(e), \psi(e), b\right)$. Since $f'_x > 0$, it requires $a\left(\omega^A(e), e\right) < b$.

For all $e \geq 0$, $a\left(\omega^A(e), e\right) < b \Rightarrow \omega^A(e) > \omega^B(e)$. Indeed, $a\left(\omega^A(e), e\right) < b$ implies $f\left(\omega^A(e), \psi(e), a\left(\omega^A(e), e\right)\right) < f\left(\omega^A(e), \psi(e), b\right)$ but $f\left(\omega^A(e), \psi(e), a\left(\omega^A(e), e\right)\right) = \underline{u} = f\left(\omega^B(e), \psi(e), b\right)$ so that $f\left(\omega^B(e), \psi(e), b\right) < f\left(\omega^A(e), \psi(e), b\right)$. Since $f'_w > 0$, it requires $\omega^A(e) > \omega^B(e)$. ■

Lemme 4 *If $\underline{u} < \tilde{u}_{(0)}$ then there exists $\tilde{e} > 0$ such that, for all $e \leq \tilde{e}$, $\omega^B(e) \leq \omega^A(e)$ and, for all $e > \tilde{e}$, $\omega^B(e) > \omega^A(e)$. If $\underline{u} \geq \tilde{u}_{(0)}$ then, for all $e \geq 0$, $\omega^B(e) \geq \omega^A(e)$.*

Proof. By definition of $\omega^A(.)$ and $\tilde{\omega}(.)$, $\underline{u} < \tilde{u}_{(0)}$ rewrites:

$$f\left(\omega^A(0), \psi(0), a\left(\omega^A(0), 0\right)\right) < f\left(\tilde{\omega}(0), \psi(0), a(\tilde{\omega}(0), 0)\right)$$

which involves $\omega^A(0) < \tilde{\omega}(0)$. $\tilde{\omega}(.)$ is differentiable for all $e \geq 0$ and $\tilde{\omega}' = -\frac{a'_e}{a'_w} < 0$ so that $\tilde{\omega}(.)$ is strictly decreasing for all $e \geq 0$. Since $\omega^A(.)$ is itself continuous and strictly increasing for all $e \geq 0$, there exists a unique $\tilde{e}$ satisfying $\tilde{\omega}(\tilde{e}) = \omega^A(\tilde{e})$. For all $e < \tilde{e}$, $\tilde{\omega}(e) > \omega^A(e) \Rightarrow a(\tilde{\omega}(e), e) > a(\omega(e), e)$. Yet, by definition of $\tilde{\omega}(.)$, $a(\tilde{\omega}(e), e) = b$ so that for all $e < \tilde{e}$, $b > a(\omega(e), e)$ and $\omega^B(e) < \omega^A(e)$ by the lemma 3. For all $e \geq \tilde{e}$, $\tilde{\omega}(e) \leq \omega^A(e) \Rightarrow a(\tilde{\omega}(e), e) \leq a(\omega(e), e)$. From which it follows that for all $e \geq \tilde{e}$, $b \leq a(\omega(e), e)$ and $\omega^B(e) \geq \omega^A(e)$.

Let us turn to the second part of the lemma. By definition of $\omega^A(.)$, for all $e \geq 0$,

$$f\left(\omega^A(e), \psi(e), a\left(\omega^A(e), e\right)\right) = \underline{u} \geq \tilde{u}_{(0)} = f(\tilde{\omega}(0), \psi(0), b)$$

while, since $\tilde{\omega}(.)$ is a strictly decreasing function, for all $e > 0$, $f(\tilde{\omega}(0), \psi(0), b) > f(\tilde{\omega}(e), \psi(e), b)$ with, by definition of $\tilde{\omega}(.)$, for all $e \geq 0$, $f(\tilde{\omega}(e), \psi(e), b) = f(\tilde{\omega}(e), \psi(e), a(\tilde{\omega}(e), e))$. Overall, for all $e \geq 0$:

$$f\left(\omega^A(e), \psi(e), a\left(\omega^A(e), e\right)\right) \geq f(\tilde{\omega}(e), \psi(e), a(\tilde{\omega}(e), e))$$

which leads to $\omega^A(e) \geq \tilde{\omega}(e)$ and $a\left(\omega^A(e), e\right) \geq a(\tilde{\omega}(e), e)$. But $a(\tilde{\omega}(e), e) = b$ so that, for all $e \geq 0$, $a\left(\omega^A(e), e\right) \geq b$ which, by the lemma 3, implies $\omega^A(e) \leq \omega^B(e)$. ■

Remark 6 $\omega^{A\prime}(\tilde{e}) < \omega^{B\prime}(\tilde{e})$.

Proof. Let us assume $\frac{d\omega^A}{de}(\tilde{e}) \geq \frac{d\omega^B}{de}(\tilde{e})$ that is

$$-\frac{\frac{\partial u^A}{\partial e}\left(\omega^A(\tilde{e}), \tilde{e}\right)}{\frac{\partial u^A}{\partial w}\left(\omega^A(\tilde{e}), \tilde{e}\right)} \geq -\frac{\frac{\partial u^B}{\partial e}\left(\omega^B(\tilde{e}), \tilde{e}\right)}{\frac{\partial u^B}{\partial w}\left(\omega^B(\tilde{e}), \tilde{e}\right)} > 0$$

For $\omega^A(\tilde{e}) = \omega^B(\tilde{e}) = \tilde{\omega}(\tilde{e}) = \tilde{w}$, it rewrites:

$$\begin{aligned}
-\frac{\frac{\partial u^A}{\partial e}(\tilde{w}, \tilde{e})}{\frac{\partial u^A}{\partial w}(\tilde{w}, \tilde{e})} &\geq -\frac{\frac{\partial u^B}{\partial e}(\tilde{w}, \tilde{e})}{\frac{\partial u^B}{\partial w}(\tilde{w}, \tilde{e})} (> 0) \\
-\frac{f'_\psi(\tilde{w}, \psi(\tilde{e}), a(\tilde{w}, \tilde{e}))\psi'(\tilde{e}) + f'_x(\tilde{w}, \psi(\tilde{e}), a(\tilde{w}, \tilde{e}))a'_e(\tilde{w}, \tilde{e})}{f'_w(\tilde{w}, \psi(\tilde{e}), a(\tilde{w}, \tilde{e})) + f'_x(\tilde{w}, \psi(\tilde{e}), a(\tilde{w}, \tilde{e}))a'_w(\tilde{w}, \tilde{e})} &\geq -\frac{f'_\psi(\tilde{w}, \psi(\tilde{e}), b)\psi'(\tilde{e})}{f'_w(\tilde{w}, \psi(\tilde{e}), b)} (> 0)
\end{aligned}$$

and, since $a(\tilde{w}, \tilde{e}) = a(\tilde{\omega}(\tilde{e}), \tilde{e}) = b$, by definition of $\tilde{\omega}(.)$:

$$-\frac{f'_\psi(\tilde{w}, \psi(\tilde{e}), b)\psi'(\tilde{e}) + f'_x(\tilde{w}, \psi(\tilde{e}), b)a'_e(\tilde{w}, \tilde{e})}{f'_w(\tilde{w}, \psi(\tilde{e}), b) + f'_x(\tilde{w}, \psi(\tilde{e}), b)a'_w(\tilde{w}, \tilde{e})} \geq -\frac{f'_\psi(\tilde{w}, \psi(\tilde{e}), b)\psi'(\tilde{e})}{f'_w(\tilde{w}, \psi(\tilde{e}), b)} (> 0)$$

and, since $f'_\psi(\tilde{w}, \psi(\tilde{e}), b)\psi'(\tilde{e}) < 0$:

$$\begin{aligned}
\frac{f'_\psi(\tilde{w}, \psi(\tilde{e}), b)\psi'(\tilde{e}) + f'_x(\tilde{w}, \psi(\tilde{e}), b)a'_e(\tilde{w}, \tilde{e})}{f'_\psi(\tilde{w}, \psi(\tilde{e}), b)\psi'(\tilde{e})} &\geq \frac{f'_w(\tilde{w}, \psi(\tilde{e}), b) + f'_x(\tilde{w}, \psi(\tilde{e}), b)a'_w(\tilde{w}, \tilde{e})}{f'_w(\tilde{w}, \psi(\tilde{e}), b)} (> 0) \\
\frac{f'_x(\tilde{w}, \psi(\tilde{e}), b)a'_e(\tilde{w}, \tilde{e})}{f'_\psi(\tilde{w}, \psi(\tilde{e}), b)\psi'(\tilde{e})} &\geq \frac{f'_x(\tilde{w}, \psi(\tilde{e}), b)a'_w(\tilde{w}, \tilde{e})}{f'_w(\tilde{w}, \psi(\tilde{e}), b)}
\end{aligned}$$

which is false. Indeed, $\frac{f'_x(\tilde{w}, \psi(\tilde{e}), b)a'_w(\tilde{w}, \tilde{e})}{f'_w(\tilde{w}, \psi(\tilde{e}), b)} > 0$ while $\frac{f'_x(\tilde{w}, \psi(\tilde{e}), b)a'_e(\tilde{w}, \tilde{e})}{f'_\psi(\tilde{w}, \psi(\tilde{e}), b)\psi'(\tilde{e})} < 0$ so that

$$\frac{f'_x(\tilde{w}, \psi(\tilde{e}), b)a'_w(\tilde{w}, \tilde{e})}{f'_w(\tilde{w}, \psi(\tilde{e}), b)} > \frac{f'_x(\tilde{w}, \psi(\tilde{e}), b)a'_e(\tilde{w}, \tilde{e})}{f'_\psi(\tilde{w}, \psi(\tilde{e}), b)\psi'(\tilde{e})}$$

Hence, $\frac{d\omega^A}{de}(\tilde{e}) < \frac{d\omega^B}{de}(\tilde{e})$. ■

Remark 7 *In equilibrium, the constraint is binding* $u(w^*, e^*) = \underline{u}$.

Proof. Suppose $u(w^*, e^*) > \underline{u}$ and consider the pair of positive deviations (dw, de) (which exists), defined by $u(w^* - dw, e^* + de) = \underline{u}$. Clearly, $S'(e^*)de - w^*(-dw) > 0$ which implies that (w^*, e^*) is not an optimum, a contradiction. ■

Lemme 8 $(w^*, e^*) \neq (\tilde{w}, \tilde{e})$.

Proof. Let us consider two pairs of marginal deviations: (d_Aw, d_Ae) defined by $\frac{d_Aw}{d_Ae} = \frac{d\omega^A}{de}(\tilde{e})$ (that is such that $du^A = 0$) and (d_Bw, d_Be) defined by $\frac{d_Bw}{d_Be} = \frac{d\omega^B}{de}(\tilde{e})$ (that is such that $du^B = 0$). Let us assume $(w^*, e^*) = (\tilde{w}, \tilde{e})$. if $S'(\tilde{e}) > \frac{d\omega^B}{de}(\tilde{e})$ then, for $d_Bw, d_Be > 0$

$$S'(\tilde{e})d_Be - d_Bw = S'(\tilde{e})d_Be - \frac{d\omega^B}{de}(\tilde{e})d_Be = \left(S'(\tilde{e}) - \frac{d\omega^B}{de}(\tilde{e})\right)d_Be > 0$$

If $\frac{d\omega^B}{de}(\tilde{e}) \geq S'(\tilde{e}) > \frac{d\omega^A}{de}(\tilde{e})$ then, for $d_A w, d_A e > 0$

$$S'(\tilde{e})\, d_A e - d_A w = S'(\tilde{e})\, d_A e - \frac{d\omega^A}{de}(\tilde{e})\, d_A e = \left(S'(\tilde{e}) - \frac{d\omega^A}{de}(\tilde{e})\right) d_A e > 0$$

If $\frac{d\omega^B}{de}(\tilde{e}) > S'(\tilde{e}) \geq \frac{d\omega^A}{de}(\tilde{e})$ then, for $d_B w, d_B e < 0$

$$S'(\tilde{e})\, d_B e - d_B w = S'(\tilde{e})\, d_B e - \frac{d\omega^B}{de}(\tilde{e})\, d_B e = \left(S'(\tilde{e}) - \frac{d\omega^B}{de}(\tilde{e})\right) d_B e > 0$$

If $\frac{d\omega^A}{de}(\tilde{e}) > S'(\tilde{e})$ then, for $d_A w, d_A e < 0$

$$S'(\tilde{e})\, d_A e - d_A w = S'(\tilde{e})\, d_A e - \frac{d\omega^A}{de}(\tilde{e})\, d_A e = \left(S'(\tilde{e}) - \frac{d\omega^A}{de}(\tilde{e})\right) d_A e > 0$$

In every case, one can find a profitable deviation from (w^*, e^*), a contradiction. Consequently, $(w^*, e^*) \neq (\tilde{w}, \tilde{e})$. ■

Remark 9 *An optimal hiring offer (w^*, e^*) exists; it satisfies $S'(e^*) = \omega'(e^*)$ and $w^* = \omega(e^*)$.*

Proof. $S'(0) \to +\infty$ excludes a corner solution. We just have to check that our assumptions are such that first order optimality conditions be sufficient optimality condition. The Lagrangian writes $L(w, e; \lambda) = S(e) - w + \lambda(u(w, e) - \underline{u})$, $\lambda > 0$. First order conditions lead to the pair (w^*, e^*) defined above. By lemma 8, $(w^*, e^*) \neq (\tilde{w}, \tilde{e})$ so that $u(.)$ is indeed differentiable for this point. The second order condition writes:

$$d^2 L(w^*, e^*; \lambda) = (dw, de) \begin{pmatrix} L''_{w^2}(w^*, e^*; \lambda) & L''_{we}(w^*, e^*; \lambda) \\ L''_{ew}(w^*, e^*; \lambda) & L''_{e^2}(w^*, e^*; \lambda) \end{pmatrix} \begin{pmatrix} dw \\ de \end{pmatrix} \leq 0$$

with $L''_{w^2}(w^*, e^*; \lambda) = \lambda u''_{w^2}(w^*, e^*)$, $L''_{we}(w^*, e^*; \lambda) = L''_{ew}(w^*, e^*; \lambda) = \lambda u''_{we}(w^*, e^*)$, and $L''_{e^2}(w^*, e^*; \lambda) = S''(e^*) + \lambda u''_{e^2}(w^*, e^*)$ so that

$$d^2 L(w^*, e^*; \lambda) = \lambda \left(\underbrace{u''_{w^2}(w^*, e^*)}_{\leq 0} dw^2 + 2\underbrace{u''_{we}(w^*, e^*)}_{\leq 0} dwde + \underbrace{u''_{e^2}(w^*, e^*)}_{\leq 0} de^2 \right) + \underbrace{S''(e^*)}_{\leq 0} de^2$$

and thus the conclusion. ■

Remark 10 *If $\underline{u} \geq \tilde{u}_{(0)}$, then $(w^*, e^*) = (w^A, e^A)$.*

Proof. By lemma 4, $\underline{u} \geq \tilde{u}_{(0)} \Rightarrow \forall e > 0, \omega^B(e) > \omega^A(e)$ so that $\omega^B(e^B) > \omega^A(e^B)$. Consequently, $S(e^B) - \omega^B(e^B) < S(e^B) - \omega^A(e^B)$. But by definition of e^A, $S(e^B) - \omega^A(e^B) \leq S(e^A) - \omega^A(e^A) = \pi^A$. Hence, $\pi^A > \pi^B$ and $(w^*, e^*) = (w^A, e^A)$. ■

Remark 11 *Let us assume $\underline{u} < \tilde{u}_{(0)}$.*

If $S'(\tilde{e}) \geq \frac{d\omega^B}{de}(\tilde{e})$ then $(w^*, e^*) = (w^A, e^A)$.

If $S'(\tilde{e}) \leq \frac{d\omega^A}{de}(\tilde{e})$ then $(w^*, e^*) = (w^B, e^B)$.

Proof. $\underline{u} < \tilde{u}_{(0)}$ ensures that $\tilde{e} > 0$.
$S'(\tilde{e}) > \frac{d\omega^B}{de}(\tilde{e}) \Rightarrow e^B > \tilde{e}$. By lemma 4, $\forall e > \tilde{e}$, $\omega^B(e) > \omega^A(e)$ so that $\omega^B(e^B) > \omega^A(e^B)$. As a result, $S(e^B) - \omega^B(e^B) < S(e^B) - \omega^A(e^B) \leq S(e^A) - \omega^A(e^A) = \pi^A$ by definition of e^A: $(w^*, e^*) = (w^A, e^A)$.
$S'(\tilde{e}) < \frac{d\omega^A}{de}(\tilde{e}) \Rightarrow e^A < \tilde{e}$. By lemma 4, $\forall e < \tilde{e}$, $\omega^B(e) < \omega^A(e)$ so that $\omega^B(e^A) < \omega^A(e^A)$. As a result, $S(e^A) - \omega^A(e^A) < S(e^A) - \omega^B(e^A) \leq S(e^B) - \omega^B(e^B) = \pi^B$ by definition of e^B: $(w^*, e^*) = (w^B, e^B)$. ■

Remark 12 *If $\underline{u} < \tilde{u}_{(0)}$ and $\omega^{B\prime}(\tilde{e}) > S'(\tilde{e}) > \omega^{A\prime}(\tilde{e})$ candidates to equilibrium (w^A, e^A) and (w^B, e^B) are such that $w^A > w^B$ and $e^A > e^B$.*

Proof. Function $\omega(.)$ is strictly increasing. We thus just have to show that $e^A > e^B$. $\frac{d\omega^B}{de}(\tilde{e}) > S'(\tilde{e}) \geq \frac{d\omega^A}{de}(\tilde{e}) \Rightarrow e^B < \tilde{e} \leq e^A$ while $\frac{d\omega^B}{de}(\tilde{e}) \geq S'(\tilde{e}) > \frac{d\omega^A}{de}(\tilde{e}) \Rightarrow e^B \leq \tilde{e} < e^A$. Hence, in every case, $e^B < e^A$ and $w^B = \omega(e^B) < w^A = \omega(e^A)$. ■

Proposition 14 *For $X \in \{A, B\}$, $\frac{\delta e^X}{\delta S'} > 0$ and $\frac{\delta w^X}{\delta S'} > 0$.*

Proof. For $X \in \{A, B\}$, $\delta S' > 0 \Rightarrow S'(e^X) + \delta S' > \omega^{X\prime}(e^X)$. Since $S'(.)$ and $\omega^{X\prime}(.)$ are respectively decreasing and increasing in e, the optimality condition $S'(e^X + \delta e^X) + \delta S' = \omega^{X\prime}(e^X + \delta e^X)$ implies $\omega^{X\prime}(e^X + \delta e^X) - S'(e^X + \delta e^X) > 0$ and thus $\delta e^X > 0$. Moreover, $\delta w^X = \omega^{X\prime}(e^X)\,\delta e^X > 0$. ■

Proposition 15 *$\delta S' > 0$ increases the attraction of identity A.*

Proof. For $X \in \{A, B\}$, $\delta \pi^X = \delta_{e^X} S(e^X) + S'(e^X)\,\delta e^X - \omega'(e^X)\,\delta e^X$ since $\omega(.)$ is not affected by $\delta S'$. $S'(e^X) = \omega'(e^X)$ implies $\delta \pi^X = \delta_{e^X} S(e^X)$. For all e, one can write $S(e) = \int_0^e S'(z)\,dz + S(0)$ and thus

$$\delta_e S(e) = \delta_e \left(\int_0^e S'(z)\,dz \right) + \delta_0 S(0) \Rightarrow \delta_e S(e) = \int_0^e \delta_z S'(z)\,dz + \delta_0 S(0)$$

By assumption, $\delta_z S'(z)$ is constant equal to $\delta S'$ so that

$$\delta_e S(e) = \delta S' \int_0^e dz + \delta_0 S(0) = \delta S'.e + \delta_0 S(0)$$

and $\delta \pi^X = \delta S'.e^X + \delta_0 S(0)$. Since $e^A > e^B$, it follows that $\delta S' > 0 \Rightarrow \delta \pi^A > \delta \pi^B$, hence the conclusion. ■

The next lemma provides a useful intermediary result. We denote $\delta_{(w,e)} u$ the variation in $u(w, e)$ resulting from $\delta \psi$ for (w, e) constant. In the steps below, provided it induces no ambiguity, we omit functions' arguments.

Lemma *If $\underline{u} < \tilde{u}_{(0)}$ then $\delta \psi > 0 \Rightarrow \delta_e \omega = -\frac{\delta_{(w,e)} u}{u'_w}$.*

Proof. By definition of $\omega(.)$, $u(\omega(e), e) = \underline{u}$ and

$$\begin{aligned} u'_w \delta_e \omega + \delta_{(w,e)} u &= \delta \underline{u} \\ \delta_e \omega &= \frac{\delta \underline{u} - \delta_{(w,e)} u}{u'_w} \end{aligned}$$

$\underline{u} < \tilde{u}_{(0)} \Rightarrow \underline{u} = u^B_{out} > u^A(0,0)$ and, since u^B_{out} is not affected by $\delta\psi$, $\delta\underline{u} = 0$ if $\delta u^A(0,0) < 0$ and $0 \leq \delta\underline{u} < \delta u^A(0,0)$ if $\delta u^A(0,0) > 0$. But $\frac{\delta_{(0,0)} u^A(0,0)}{\delta\psi} < 0$ so the conclusion. ■

In the statements below, we use the following short writings: u^X for $u^X(w^X, e^X)$, f^X for $f(w^X, \psi(e^X), x(w^X, e^X))$, a^X for $a(w^X, e^X)$, ω^X for $\omega^X(e^X)$ with $X \in \{A, B\}$.

Proposition 16 *For $X \in \{A, B\}$:*

(1) if marginal rates of substitution of effort for wage are non decreasing in work strain then, all other things remaining equale, $\delta\psi > 0 \Rightarrow \delta e^X \leq 0$;

(2) if income effects dominate substitution effects then, all other things remaining equale, $\delta\psi > 0 \Rightarrow \delta w^X > 0$.

Proof. For $X \in \{A, B\}$, by definition of e^X, $\omega^{X\prime}(e^X) = S'(e^X)$, which entails

$$\begin{aligned} \delta_{e^X} \omega^{X\prime} + \omega^{X\prime\prime} \delta e^X &= S^{X\prime\prime} \delta e^X \\ \delta e^X &= \frac{\delta_{e^X} \omega^{X\prime}}{S^{X\prime\prime} - \omega^{X\prime\prime}} \end{aligned}$$

the assumption that marginal rates of substitution of effort for wage are non decreasing in work strain means that, for all e, $\delta\psi > 0 \Rightarrow \delta_e \omega^{X\prime} \geq 0$. Furthermore, $S'' - \omega'' < 0$, so that $\delta\psi > 0 \Rightarrow \delta e^X < 0$. For $X \in \{A, B\}$, $w^X = \omega^X(e^X)$ and $\delta w^X = \omega'(e^X) \delta e^X + \delta_{e^X} \omega(e^X)$ with, for $\delta\psi > 0$, by previous compagnon lemma,

$$\delta_e \omega = -\frac{\delta_{(w,e)} u}{u'_w}$$

where $\delta_{(w,e)} u$ is the variation in utility resulting, all other things remaining equale, from $\delta\psi > 0$. It is obviously the case that $\delta_{(w,e)} u < 0$ so that $\delta_e \omega > 0$. We eventually have $\omega'(e^X) \delta e^X \leq 0$, the substitution effect of $\delta\psi > 0$, on the one hand, and $\delta_{e^X} \omega(e^X) > 0$, its income effect, on the other. Assuming the latter dominates the former, $\delta\psi > 0 \Rightarrow \delta w^X > 0$. ■

Remark 17 *$\delta\psi > 0$ increases the attraction of identity A if and only if $-\frac{f^{B\prime}_\psi}{u^{B\prime}_w} > -\frac{f^{A\prime}_\psi}{u^{A\prime}_w} (> 0)$.*

Proof. For $X \in \{A, B\}$, profit writes $\pi^X = S(e^X) - \omega^X(e^X)$ so that $\delta\pi^X = \delta S(e^X) - \delta\omega^X(e^X)$ with $\delta S(e^X) = S'(e^X) \delta e^X$ and $\delta\omega^X(e^X) = \omega^{X\prime}(e^X) \delta e^X + \delta_{e^X} \omega^X(e^X)$. Thus

$$\delta\pi^X = \left(S'(e^X) - \omega^{X\prime}(e^X)\right) \delta e^X - \delta_{e^X} \omega^X(e^X)$$

Yet, by definition of e^X, $S'\left(e^X\right) = \omega^{X\prime}\left(e^X\right)$ so that $\delta\pi^X = -\delta_{e^X}\omega^X\left(e^X\right)$. By the last lemma, for $\delta\psi > 0$,

$$\delta_e^X\omega = -\frac{\delta_{(w,e)}u}{u'_w} = -\frac{f'_\psi}{u'_w}\delta\psi$$

and, for $X \in \{A, B\}$, $\frac{\delta\pi^X}{\delta\psi} = \frac{f_\psi^{X\prime}}{u_w^{X\prime}} < 0$. ■

Proposition 18 *Suppose $\psi'' \geq 0$ and, for all $(w,e) \in \mathbb{R}_+^2$, $a(w,e) = \check{a}(w, \psi(e))$. Then $\delta\psi > 0$ increases identity A attraction.*

Proof. With this added behavioral assumption, remark 17 rewrites: $\delta\psi > 0$ increases the attraction of identity A if and only if $-\frac{f_\psi^{B\prime}}{u_w^{B\prime}} > -\frac{f_\psi^{A\prime}+f_x^{A\prime}\check{x}'_\psi}{u_w^{A\prime}}$ (> 0).

We have $\omega^{B\prime} > \omega^{A\prime}$. This rewrites

$$\begin{aligned}
-\frac{f_\psi^{B\prime}\psi^{B\prime}}{u_w^{B\prime}} &> -\frac{f_\psi^{A\prime}\psi^{A\prime} + f_x^{A\prime}\check{x}_\psi^{A\prime}\psi^{A\prime}}{u_w^{A\prime}} (> 0) \\
-\frac{f_\psi^{B\prime}}{u_w^{B\prime}} &> -\left(\frac{\psi^{A\prime}}{\psi^{B\prime}}\right)\frac{f_\psi^{A\prime} + f_x^{A\prime}\check{x}_\psi^{A\prime}}{u_w^{A\prime}} (> 0)
\end{aligned}$$

since $e^A > e^B$, for $\psi'' \geq 0$, $\psi^{A\prime} \geq \psi^{B\prime}$ and therefore, $\frac{\psi^{A\prime}}{\psi^{B\prime}} \geq 1$. As a consequence

$$-\left(\frac{\psi^{A\prime}}{\psi^{B\prime}}\right)\frac{f_\psi^{A\prime} + f_x^{A\prime}\check{x}_\psi^{A\prime}}{u_w^{A\prime}} \geq -\frac{f_\psi^{A\prime} + f_x^{A\prime}\check{x}_\psi^{A\prime}}{u_w^{A\prime}} (> 0)$$

and thus

$$-\frac{f_\psi^{B\prime}}{u_w^{B\prime}} > -\frac{f_\psi^{A\prime} + f_x^{A\prime}\check{x}_\psi^{A\prime}}{u_w^{A\prime}} (> 0)$$

which is precisely the condition which garantees $\delta\pi^A > \delta\pi^B$ in the case $\check{x}_\psi^{A\prime} \neq 0$. Hence, $\delta\psi > 0$ indeed increases identity A attraction. ■

Proposition 19 *Suppose $0 < \delta a \leq \frac{u_{out}^B - u_0^A(0,0)}{f'_x(0,\psi(0),a_0(0,0))}$. Then hiring is selective and favourable to type 1 applicants if and only if*

$$\pi_0^B - \pi_0^A < \frac{f_x^{A_0\prime}}{u_w^{A_0\prime}}\delta a$$

where $u^{A_0} = f\left(w_0^A, \psi\left(e_0^A\right), a^{A_0}\right)$.

Proof. As we have seen already, for small variations δe,

$$\delta\pi = S'(e^*)\,\delta e - \left(\omega'(e^*)\,\delta e + \delta_{e^*}\omega(e^*)\right) = -\delta_{e^*}\omega(e^*)$$

since $S'(e^*) = \omega'(e^*)$ by definition of e^*. Let us make $\delta_{e^A}\omega^A\left(e^A\right)$ explicit, i.e. the variation of ω^A

assuming that e remain constant equal to e^A. By definition of $\omega^A(.)$, $f\left(\omega^A(e), \psi(e), a\left(\omega^A(e), e\right)\right) = \underline{u}$

$$f_w^{A\prime}\delta_e\omega^A + f_x^{A\prime}.\left(a'_w\delta_e\omega^A + \delta a\right) = \delta\underline{u}$$

where $\delta \underline{u} = \max\left\{f'_x\left(0,\psi\left(0\right),a_0\left(0,0\right)\right)\delta a + \left(u_0^A\left(0,0\right) - u^B_{out}\right);0\right\} = 0$ since $0 < \delta a \leq \frac{u^B_{out} - u_0^A(0,0)}{f'_x(0,\psi(0),a_0(0,0))}$.
Thus

$$\delta_e \omega^A = -\frac{f_x^{A\prime}}{f_w^{A\prime} + f_x^{A\prime} a'_w}\delta a < 0$$

Consequently, $\delta \pi^A = \pi_1^A - \pi_0^A = -\delta_{e_0^A}\omega\left(e_0^A\right) > 0$ *i.e.* assuming the employer targets identity A, $\delta a > 0$ entails a gain in profit. Moreover, by definition of $\omega^B\left(.\right)$, $f\left(\omega^B\left(e\right),\psi\left(e\right),b\right) = \underline{u}$

$$f_w^{B\prime}\delta_e \omega^B = 0 \Leftrightarrow \delta_e \omega^B = 0$$

so that $\delta \pi^B = \pi_1^B - \pi_0^B = 0$.

Though, for the hiring to be selective, it must be the case that $\pi_0^B = \pi_1^B < \pi_1^A$ *i.e.*, $\pi_0^B < \pi_0^A + \delta_{e_0^A}\pi^A$

$$\pi_0^B - \pi_0^A < \delta_{e_0^A}\pi^A = \frac{f_x^{A_0\prime}}{f_w^{A_0\prime} + f_x^{A_0\prime} a_w^{A_0\prime}}\delta a$$

■

Numerical example

$f\left(w,\psi,x\right) = w - \psi + x$, $\psi\left(e\right) = \psi_2 e^2 + \psi_0$, $\psi_2 > 1$, $a\left(w,e\right) = w + e^2 + a$, $a < b$. Consequently:

$u^A\left(w,e\right) = 2w + \left(1-\psi_2\right)e^2 + \left(a - \psi_0\right)$ $\quad u^B\left(w,e\right) = w - \psi_2 e^2 + \left(b - \psi_0\right)$

Assumptions:

$u^A\left(0,0\right) = a - \psi_0 < \underline{u}$ $\qquad u^B\left(0,0\right) = b - \psi_0 < \underline{u}$

Remark: $b > a \Rightarrow u^B\left(0,0\right) > u^A\left(0,0\right)$. We assume $\underline{u} = \frac{5}{4}$.

$\omega^A\left(e\right) = \frac{1}{2}\left(\left(\psi_2 - 1\right)e^2 + \underline{u} - \left(a - \psi_0\right)\right)$ $\quad \omega^B\left(e\right) = \psi_2 e^2 + \underline{u} - \left(b - \psi_0\right)$

$$\tilde{e} = \sqrt{\left(\frac{1}{1+\psi_2}\right)\left(2\left(b-\psi_0\right) - \left(a - \psi_0\right) - \underline{u}\right)}$$

$$S\left(e\right) = 2\sigma e^{\frac{1}{2}} + \sigma_0 e$$

Reference numerical values:

a	b	ψ_0	ψ_2	$\underline{u}$	$u^A\left(0,0\right)$	$u^B\left(0,0\right)$	$\tilde{e}$
0	1	0	2	$\frac{5}{4}$	0	1	$\frac{1}{2}$

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